Thinkwell's Placement Test 3 Answer Key

If you answered 7 or more Test 3 questions correctly, we recommend Thinkwell's College Algebra (Algebra 2). If you answered fewer than 7 Test 3 questions correctly, we recommend Thinkwell's Intermediate Algebra (Algebra 1).

1. **Answer:** $\mathbf{x = -\frac{1}{6}}$ **or** $\mathbf{x = -\frac{7}{6}}$

Explanation

Set up the two equations, then solve each equation.

$$|6x+4| = 3 \Rightarrow$$

$$\begin{aligned} 6x+4 &= 3 \quad \text{or} \quad & 6x+4 &= -3 \\ 6x &= -1 & 6x &= -7 \\ x &= -\frac{1}{6} & x &= -\frac{7}{6} \end{aligned}$$

Check.

$$x = -\frac{1}{6} \Rightarrow \left|6\left(-\frac{1}{6}\right)+4\right| = |-1+4| = 3 \text{ (checks)} \qquad x = -\frac{7}{6} \Rightarrow \left|6\left(-\frac{7}{6}\right)+4\right| = |-7+4| = 3 \text{ (checks)}$$

This concept is covered in Thinkwell's Intermediate Algebra lecture "Solving Absolute Value Equations."

2. **Answer:** $\mathbf{x^3 - 5x^2 + 7x - 2}$

Explanation

The expression within the 2nd set of parentheses contains like terms. So, simplify that expression first, and then multiply by using the Distributive Property. Here, subtraction is written as addition of a negative number in order to keep the signs straight when using the Distributive Property.

$$(x-2)(x^2-4x+1+x)$$

$$(x-2)(x^2-3x+1) \qquad \textit{Combine like terms.}$$

$$(x+(-2))(x^2+(-3x)+1) \qquad \textit{Write subtraction as addition of a negative.}$$

$$x(x^2+(-3x)+1)+(-2)(x^2+(-3x)+1) \qquad \textit{Distribute.}$$

$$x(x^2)+x(-3x)+x(1)+(-2)(x^2)+(-2)(-3x)+(-2)(1) \qquad \textit{Distribute.}$$

$$x^3+(-3x^2)+x+(-2x^2)+6x+(-2) \qquad \textit{Multiply.}$$

$$x^3-3x^2+x-2x^2+6x-2 \qquad \textit{Write addition of a negative as subtraction.}$$

$$x^3-5x^2+7x-2 \qquad \textit{Combine like terms.}$$

This concept is covered in Thinkwell's Intermediate Algebra lecture "Multiplying Big Products."

3. **Answer: 2*s* + 5, R 12**

Explanation

Set up the rational expression as long division where the numerator is the dividend and the denominator is the divisor.

$$\frac{2s^2-s-3}{s-3} = s-3\overline{)2s^2-s-3}$$

Complete the long division.

$$\begin{array}{r} 2s+5 \\ s-3\overline{)2s^2-s-3} \end{array}$$

$$-(2s^2-6s) \qquad \textit{Multiply 2s by } s-3 \textit{ and subtract.}$$

$$5s-3 \qquad \textit{Bring down the } -3.$$

$$-(5s-15) \qquad \textit{Multiply 5 by } s-3 \textit{ and subtract.}$$

$$12 \qquad \textit{The remainder is 12.}$$

Thus, the quotient is $2s+5$, R 12.

This concept is covered in Thinkwell's Intermediate Algebra lecture "Using Long Division with Polynomials."

4. **Answer:** $\mathbf{\frac{5x-11}{x^2-1}}$

<u>Explanation</u>

Begin by factoring the denominators, if possible. $\frac{5}{x+1}-\frac{6}{x^2-1}=\frac{5}{x+1}-\frac{6}{(x+1)(x-1)}$

Before the rational expressions can be subtracted, they must first be written as equivalent rational expressions with a common denominator. So, find the common denominator. Since the denominator of the first rational expression is $x+1$ and the denominator of the second rational expression is $(x+1)(x-1)$, the common denominator is also $(x+1)(x-1)$. Since the denominator of the second rational expression is the common denominator, that expression does not need to be rewritten.

The first rational expression needs to be written as an equivalent rational expression where the denominator is $(x+1)(x-1)$. So, multiply the first rational expression by $\frac{x-1}{x-1}$ and then subtract.

$$\frac{5}{x+1}-\frac{6}{(x+1)(x-1)}$$

$$\frac{5}{x+1}\cdot\frac{x-1}{x-1}-\frac{6}{(x+1)(x-1)}$$ *Multiply the first rational expression by x − 1 over itself.*

$$\frac{5(x-1)}{(x+1)(x-1)}-\frac{6}{(x+1)(x-1)}$$ *Multiply the numerators and multiply the denominators.*

$$\frac{5(x-1)-6}{(x+1)(x-1)}$$ *Subtract the 2nd numerator from the 1st numerator.*

$$\frac{5x-5-6}{x^2-1}$$ *Distribute 5 in the numerator. FOIL the denominator.*

$$\frac{5x-11}{x^2-1}$$ *Combine like terms in the numerator.*

This concept is covered in Thinkwell's Intermediate Algebra lecture "Adding and Subtracting Rational Expressions."

5. **Answer:** $\mathbf{x = 13}$

<u>Explanation</u>

Since one side of the equation is a radical, begin by squaring both sides to remove the radical.

$$\sqrt{4x-3}=x-6$$
$$\left(\sqrt{4x-3}\right)^2=(x-6)^2$$
$$4x-3=x^2-12x+36$$

The resulting equation is quadratic. So, set the equation equal to 0 by bringing all terms to either side and then solve by factoring or by using the quadratic formula.

$$4x-3=x^2-12x+36$$
$$0=x^2-12x+36-4x+3$$
$$0=x^2-16x+39$$
$$0=(x-13)(x-3)$$
$$x-13=0 \quad \text{or} \quad x-3=0$$
$$x=13 \qquad\qquad x=3$$

Check that both solutions satisfy the equation.

$\boxed{x=13}$	$\boxed{x=3}$
$\sqrt{4(13)-3}=(13)-6$	$\sqrt{4(3)-3}=(3)-6$
$\sqrt{49}=7$	$\sqrt{9}=-3$
$7=7$	$3\neq-3$

So, $x=3$ is not a solution since it does not satisfy the equation. The solution is $x=13$.

This concept is covered in Thinkwell's Intermediate Algebra lecture "Solving an Equation Containing a Radical."

6. **Answer:** $\boldsymbol{y = -4x - 25}$

Explanation

The slope, m, and y-intercept, b, of a line must be known in order to write the equation of that line in slope-intercept form, $y = mx + b$. Begin by using the coordinates from the two given points to find the slope of the line. Then use the coordinates from either of the given points and the slope to find the y-intercept. Once the slope and y-intercept are known, substitute those values into slope-intercept form, $y = mx + b$, to write the equation of the line.

Find the slope. Substitute the coordinates from the two points into the slope formula. Let $(-6,\ -1)$ be (x_1, y_1) and let $(-8, 7)$ be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-1)}{-8-(-6)} = \frac{8}{-2} = -4$$

Find the y-intercept. Substitute the slope and the x- and y-coordinates from *either* given point into slope-intercept form and solve for b. Here, the coordinates from $(-6,\ -1)$ are used to find b.

$$\begin{aligned} y &= mx + b && \textit{Slope-Intercept Form} \\ -1 &= -4(-6) + b && \textit{Substitute } -4 \textit{ for } m, -6 \textit{ for } x, \textit{and } -1 \textit{ for } y. \\ -1 &= 24 + b && \textit{Multiply.} \\ b &= -25 && \textit{Subtract 24 from each side.} \end{aligned}$$

Write the equation. Substitute the m and b values into slope-intercept form, $y = mx + b$. Note that values are *not* substituted in for x and y when the equation of the line is written.

$$\begin{aligned} y &= mx + b && \textit{Slope-Intercept Form} \\ y &= -4x + (-25) && \textit{Substitute } -4 \textit{ for } m \textit{ and } -25 \textit{ for } b. \\ y &= -4x - 25 && \textit{Write addition of a negative number as subtraction.} \end{aligned}$$

This concept is covered in Thinkwell's Intermediate Algebra lecture "Writing an Equation Given Two Points."

7. **Answer:** $\boldsymbol{y = -\frac{3}{4}x + 1}$

Explanation

Find the slope. The slope of $y = \frac{4}{3}x - 1$ is $\frac{4}{3}$. The slopes of perpendicular lines are opposite reciprocals.

So, the slope of any line that is perpendicular to $y = \frac{4}{3}x - 1$ is $-\frac{3}{4}$. Thus, $m = -\frac{3}{4}$.

Find the y-intercept. Substitute the slope, $m = -\frac{3}{4}$, and the x- and y-coordinates from the given point, $(4, -2)$, into slope-intercept form and solve for b.

$$\begin{aligned} y &= mx + b && \textit{Slope-Intercept Form} \\ -2 &= -\frac{3}{4}(4) + b && \textit{Substitute } -3/4 \textit{ for } m, 4 \textit{ for } x, \textit{and } -2 \textit{ for } y. \\ -2 &= -3 + b && \textit{Multiply.} \\ b &= 1 && \textit{Add 3 to both sides.} \end{aligned}$$

Write the equation. Substitute $m = -3/4$ and $b = 1$ into slope-intercept form.

$$\begin{aligned} y &= mx + b && \textit{Slope-Intercept Form} \\ y &= -\frac{3}{4}x + 1 && \textit{Substitute } -3/4 \textit{ for } m \textit{ and 1 for } b. \end{aligned}$$

This concept is covered in Thinkwell's Intermediate Algebra lecture "Slope for Parallel and Perpendicular Lines."

8. **Answer: (4, 3)**

Explanation

The 2nd equation, $x = 19 - 5y$, is solved for x. So, $19 - 5y$ can be substituted into the 1st equation for x.

$$\boxed{x} + 4y = 16$$
$$\nwarrow$$
$$x = \boxed{19 - 5y}$$

After $19 - 5y$ is substituted for x, the resulting equation includes only one variable, y. Simplify the resulting equation and then solve for y.

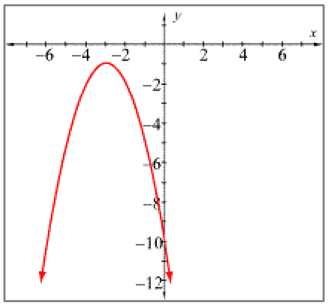
$$\begin{aligned} x + 4y &= 16 \\ (19 - 5y) + 4y &= 16 && \textit{Substitute } 19 - 5y \textit{ for } x. \\ 19 - y &= 16 && \textit{Combine like terms.} \\ -y &= -3 && \textit{Subtract 19 from each side.} \\ y &= 3 && \textit{Divide each side by } -1. \end{aligned}$$

Find the x-coordinate of the solution. Substitute $y = 3$ into *either* of the original equations to find the value of x. Here, 3 will be substituted for y in the equation $x = 19 - 5y$.

$$\begin{aligned} x &= 19 - 5y \\ x &= 19 - 5(3) && \textit{Substitute 3 for } y. \\ x &= 19 - 15 && \textit{Multiply.} \\ x &= 4 && \textit{Subtract.} \end{aligned}$$

So, the system's solution is $(4, 3)$.

This concept is covered in Thinkwell's Intermediate Algebra lecture "Solving a System by Substitution."

9. **Answer:** 

Explanation

Determine the general shape of the function. The given function, $y=-(x+3)^2-1$, is in the vertex form of a quadratic equation, $y=a(x-h)^2+k$. A vertex form equation describes a graph that is a parabola. The parabola opens either upwards or downwards.
So, graph the given function by finding changes in direction, steepness, and shifts as compared to the general form of a quadratic function, $y=x^2$, which is a parabola with vertex at (0, 0) that opens upwards.
Determine the direction. The direction that a parabola opens is given by the sign of a in the vertex form equation, $y=a(x-h)^2+k$. If a is positive, then the parabola opens upwards. If a is negative, then the parabola opens downwards. In the given function, $y=-(x+3)^2-1$, $a=-1$. Therefore, a is negative and so the parabola opens downwards.
Determine the steepness. The steepness of a parabola is given by the absolute value of a in the vertex form equation, $y=a(x-h)^2+k$. If $|a|>1$, then the parabola is wider than the parabola formed by $y=x^2$. If $|a|<1$, then the parabola is steeper than the parabola formed by $y=x^2$. If $|a|=1$, then the parabola is the same width as the parabola formed by $y=x^2$. In the given function, $|a|=1$. Therefore, the parabola has the same width as $y=x^2$.
Determine any shifts. The number of units that a parabola is shifted from the origin is given by the values of h and k in the vertex form equation, $y=a(x-h)^2+k$. The value of h gives the horizontal shift (left or right) and the value of k gives the vertical shift (up or down). If h is positive, then the parabola is shifted h units to the left. If h is negative, then the parabola is shifted $|h|$ units to the right. Positive values of k indicate a shift k units up and negative values of k indicate a shift of $|k|$ units down.
Identify the values of h and k in the given function, $y=-(x+3)^2-1$.

$$y=-(x+3)^2-1 \Rightarrow y=-(x-(-3))^2+(-1)$$

So, $h=-3$ and $k=-1$. Therefore, the parabola is shifted 3 units to the right and 1 unit down.

This concept is covered in Thinkwell's Intermediate Algebra lecture "Graphing Quadratics Using Pattern."

10. **Answer: E-None of the above**
Explanation
A. The discriminant is the value of $b^2 - 4ac$ where a, b, and c are the coefficients and constant term from a quadratic equation in standard form, $ax^2 + bx + c = 0$. If the discriminant is 0, $b^2 - 4ac = 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with exactly one x-intercept. If the discriminant is positive, $b^2 - 4ac > 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with exactly two x-intercepts. If the discriminant is negative, $b^2 - 4ac < 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with no x-intercepts. The given parabola has exactly one x-intercept, the parabola's vertex. Therefore, the discriminant cannot be positive.
B. The constant term of a quadratic function is the parabola's y-intercept. So, if the constant term is zero, then the parabola must intersect the y-axis at 0. The given parabola's y-intercept is at 2, so the constant term is 2.
C. The given parabola opens upwards. Therefore, the coefficient of x^2 must be positive, not negative.
D. The given parabola has exactly one x-intercept. Therefore, the discriminant cannot be negative.

This concept is covered in Thinkwell's Intermediate Algebra lecture "Using the Discriminant to Graph Parabolas."