Thinkwell's Placement Test 4 Answer Key

If you answered 7 or more Test 4 questions correctly, we recommend Thinkwell's Precalculus. If you answered fewer than 7 Test 4 questions correctly, we recommend Thinkwell's College Algebra (Algebra 2).

1. **Answer: –3**

Explanation

The given function $f(x)$ is a piecewise function.

$$f(x)=\begin{cases}-2x \text{ if } x<-2\\ 5x+7 \text{ if } -2\le x\le 4\\ -7x \text{ if } x>4\end{cases}$$

The domain is divided into three parts: x-values less than -2, x-values between -2 and 4 (including -2 and 4), and x-values greater than 4. For all x-values less than -2, the function used is $f(x)=-2x$. For all x-values between -2 and 4, the function used is $f(x)=5x+7$. And, for all values greater than 4, the function used is $f(x)=-7x$. Therefore, the second function, $f(x)=5x+7$, is used to evaluate $f(-2)$, since -2 is in the domain of this piece of the function. $f(-2)=5(-2)+7=-3$

Thus, $f(-2)=-3$.

This concept is covered in Thinkwell's College Algebra lecture "Evaluating Piecewise-Defined Functions for Given Values."

2. **Answer: vertex (3, 2); *x* = 3**

Explanation

The vertex of a parabola defined by a quadratic equation, $y=ax^2+bx+c$, is located at the point (h,k), where $h=-\frac{b}{2a}$ and $k=ah^2+bh+c$. So, to find the coordinates of a parabola's vertex, first find the value of h, and then substitute that h-value into the given equation to find the value of k. The given equation, $y=5x^2-30x+47$, is in the standard form of a quadratic equation, $y=ax^2+bx+c$, where $a=5$, $b=-30$, and $c=47$.

Use the values of a and b to find the x-coordinate of the vertex, h.

$$h=-\frac{b}{2a}=-\frac{-30}{2(5)}=3$$

So, the vertex's x-coordinate is 3.

Now substitute 3 into the given equation to find the value of k, which is the vertex's y-coordinate.

$$\begin{aligned} y &= 5x^2-30x+47 \\ &= 5(3)^2-30(3)+47 && \textit{Substitute 3 for x.} \\ &= 5(9)-30(3)+47 && \textit{Evaluate the power.} \\ &= 2 && \textit{Simplify.} \end{aligned}$$

So, the vertex's y-coordinate is2.

Therefore, the parabola's vertex is at $(3, 2)$.

The axis of symmetry of a vertically opening parabola is a vertical line through the parabola's vertex. Since it is the x-variable that is squared in the given equation, the parabola opens vertically. Therefore, the axis of symmetry is the vertical line that passes through the vertex, $(3, 2)$. The equation of the vertical line that passes through $(3, 2)$ is $x=3$. Thus, the axis of symmetry is the line $x=3$.

This concept is covered in Thinkwell's College Algebra lecture "Determining Information about a Parabola from Its Equation."

3. **Answer:** $\boldsymbol{x=-\frac{1}{32}}$ **or** $\boldsymbol{x=1}$

Explanation

Notice that the given equation, $2x^{\frac{2}{5}}-x^{\frac{1}{5}}-1=0$, can be written in quadratic form if a variable is substituted for $x^{\frac{1}{5}}$. Let $g=x^{\frac{1}{5}}$. Substitute g into the equation and then solve the resulting quadratic equation for g by factoring (or by using the quadratic formula). After the new equation is solved for g, substitute $x^{\frac{1}{5}}$ for g and solve for x.

$$\begin{aligned} 2x^{\frac{2}{5}}-x^{\frac{1}{5}}-1&=0 \\ 2x^{\frac{1}{5}\cdot 2}-x^{\frac{1}{5}}-1&=0 && \textit{Write the exponent 2/5 as a product where one factor is 1/5.} \\ 2\left(x^{\frac{1}{5}}\right)^2-x^{\frac{1}{5}}-1&=0 && \textit{Product of Powers Property} \\ 2g^2-g-1&=0 && \textit{Substitute } g \textit{ for } x^{\frac{1}{5}}. \\ (2g+1)(g-1)&=0 && \textit{Factor.} \end{aligned}$$

$$\begin{array}{lcll} 2g+1=0 & \text{or} & g-1=0 & \textit{Zero Factor Property} \\ g=-1/2 & & g=1 & \textit{Solve for g.} \\ x^{\frac{1}{5}}=-1/2 & \text{or} & x^{\frac{1}{5}}=1 & \textit{Substitute } x^{\frac{1}{5}} \textit{ for g.} \\ x=-1/32 & & x=1 & \textit{Solve for x.} \end{array}$$

This concept is covered in Thinkwell's College Algebra lecture "Solving Fancy Quadratics."

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4. **Answer:** $\mathbf{y = 9}$

Explanation

Square each side and simplify.

$$\sqrt{2y+7}=\sqrt{2y-9}+2$$

$$\left(\sqrt{2y+7}\right)^2=\left(\sqrt{2y-9}+2\right)^2 \qquad \textit{Square both sides.}$$

$$2y+7=\left(\sqrt{2y-9}+2\right)\left(\sqrt{2y-9}+2\right) \qquad \textit{Expand the power on the right side.}$$

$$2y+7=(2y-9)+4\sqrt{2y-9}+4 \qquad \textit{FOIL}$$

$$2y+7=2y-5+4\sqrt{2y-9} \qquad \textit{Combine like terms.}$$

Now isolate the remaining radical and then square both sides again.

$$12=4\sqrt{2y-9} \qquad \textit{Subtract 2y from each side and add 5 to each side.}$$

$$3=\sqrt{2y-9} \qquad \textit{Divide each side by 4.}$$

$$(3)^2=\left(\sqrt{2y-9}\right)^2 \qquad \textit{Square both sides.}$$

$$9=2y-9 \qquad \textit{Simplify the powers.}$$

$$18=2y \qquad \textit{Add 9 to each side.}$$

$$y=9 \qquad \textit{Divide each side by 2.}$$

This concept is covered in Thinkwell's College Algebra lecture "Solving an Equation with Two Radicals."

5. **Answer:** $\dfrac{\mathbf{w^2-2w-1}}{\mathbf{w^2+3w+7}}$

Explanation

Factor the denominators.

$$\frac{\dfrac{1}{w^2-3w-4}+\dfrac{1}{w+3}}{\dfrac{1}{w-4}-\dfrac{1}{w^2+4w+3}}=\frac{\dfrac{1}{(w-4)(w+1)}+\dfrac{1}{w+3}}{\dfrac{1}{w-4}-\dfrac{1}{(w+3)(w+1)}}$$

The common denominator of the four terms is $(w-4)(w+1)(w+3)$, so multiply the expression by $\dfrac{(w-4)(w+1)(w+3)}{(w-4)(w+1)(w+3)}$.

$$\frac{(w-4)(w+1)(w+3)}{(w-4)(w+1)(w+3)}\left(\frac{\dfrac{1}{(w-4)(w+1)}+\dfrac{1}{w+3}}{\dfrac{1}{w-4}-\dfrac{1}{(w+3)(w+1)}}\right)=\frac{\dfrac{1(w-4)(w+1)(w+3)}{(w-4)(w+1)}+\dfrac{1(w-4)(w+1)(w+3)}{w+3}}{\dfrac{1(w-4)(w+1)(w+3)}{w-4}-\dfrac{1(w-4)(w+1)(w+3)}{(w+3)(w+1)}}$$

$$=\frac{(w+3)+(w-4)(w+1)}{(w+1)(w+3)-(w-4)}$$

$$=\frac{w+3+w^2-3w-4}{w^2+4w+3-w+4}$$

$$=\frac{w^2-2w-1}{w^2+3w+7}$$

This concept is covered in Thinkwell's College Algebra lecture "Rewriting Complex Fractions."

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6. **Answer:** $\mathbf{\dfrac{1 \pm \sqrt{1-k^2q^2}}{k}}$

Explanation

Multiply both sides of the equation by the expression in the denominator, p^2+q^2. Then, set the equation equal to 0.

$$k = \frac{2p}{p^2+q^2}$$

$$k\left(p^2+q^2\right) = \frac{2p}{p^2+q^2}\left(p^2+q^2\right) \quad \textit{Multiply both sides by } p^2+q^2.$$

$$kp^2+kq^2 = 2p \quad \textit{Simplify.}$$

$$kp^2-2p+kq^2 = 0 \quad \textit{Set equal to zero.}$$

Notice that the equation is quadratic in terms of *p*, $kp^2-2p+kq^2=0$. So, use the quadratic formula to solve for *p*.

Identify the values of *a*, *b*, and *c*. $kp^2-2p+kq^2=0 \quad \Rightarrow \quad a=k,\ b=-2, \text{ and } c=kq^2$

$$p = \frac{-(-2) \pm \sqrt{(-2)^2-4(k)\left(kq^2\right)}}{2(k)} \quad \textit{Substitute } k, -2, \textit{ and } kq^2 \textit{ for } a, b, \textit{ and } c \textit{ in the quadratic formula.}$$

$$= \frac{2 \pm \sqrt{4-4k^2q^2}}{2k} \quad \textit{Simplify.}$$

$$= \frac{2 \pm \sqrt{4\left(1-k^2q^2\right)}}{2k} \quad \textit{Factor the radicand.}$$

$$= \frac{2 \pm 2\sqrt{1-k^2q^2}}{2k} \quad \textit{Simplify the radical.}$$

$$= \frac{1 \pm \sqrt{1-k^2q^2}}{k} \quad \textit{Remove the factor of 2.}$$

This concept is covered in Thinkwell's College Algebra lecture "Solving for a Squared Variable."

7. **Answer: –2*x* + 1, R(2)**

Explanation

Since the denominator does not include an *x*-term, add the placeholder 0*x* to the denominator. Use long division to divide.

$$\frac{6x^3-3x^2-2x+3}{-3x^2+1} = \frac{6x^3-3x^2-2x+3}{-3x^2+0x+1}$$

$$\begin{array}{r} -2x+1 \\ -3x^2+0x+1 \,\big)\, \overline{6x^3-3x^2-2x+3} \\ -\left(6x^3+0x^2-2x\right) \\ \hline -3x^2+0x+3 \\ -\left(-3x^2+0x+1\right) \\ \hline 2 \end{array}$$

Thus, the quotient is $-2x+1$, R(2).

This concept is covered in Thinkwell's College Algebra lecture "Long Division: Another Example."

8. **Answer: *x* = –4 with multiplicity three; *x* = 3 with multiplicity one**

Explanation

Since $x=-4$ is a zero, $x+4$ is a factor. Use synthetic division to divide the polynomial by the factor $x+4$.

$$\begin{array}{r|rrrrr} -4 & 1 & 9 & 12 & -80 & -192 \\ & & -4 & -20 & 32 & 192 \\ \hline & 1 & 5 & -8 & -48 & 0 \end{array}$$

So, $\dfrac{x^4+9x^3+12x^2-80x-192}{x+4} = x^3+5x^2-8x-48$.

It follows that $x^4+9x^3+12x^2-80x-192 = (x+4)\left(x^3+5x^2-8x-48\right)$.

Since $x=3$ is a zero, $x-3$ is a factor. Use synthetic division to divide $x^3+5x^2-8x-48$ by $x-3$.

$$\begin{array}{r|rrrr} 3 & 1 & 5 & -8 & -48 \\ & & 3 & 24 & 48 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

So, $\dfrac{x^3+5x^2-8x-48}{x-3} = x^2+8x+16$.

It follows that $x^4+9x^3+12x^2-80x-192 = (x+4)\left(x^3+5x^2-8x-48\right) = (x+4)(x-3)\left(x^2+8x+16\right)$.

Factor $x^2+8x+16$. $(x+4)(x-3)\left(x^2+8x+16\right) = (x+4)(x-3)(x+4)(x+4) = (x+4)^3(x-3)$

Thus, the zeros are $x=-4$ with multiplicity three and $x=3$ with multiplicity one.

This concept is covered in Thinkwell's College Algebra lecture "Finding the Zeros of a Polynomial from Start to Finish."

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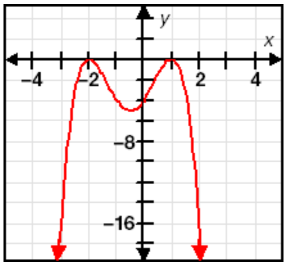
9. **Answer:** 

Explanation

Expand the polynomial.

$$\begin{aligned} f(x) &= -(x+2)^2(x-1)^2 \\ &= -\left(x^2+4x+4\right)\left(x^2-2x+1\right) \\ &= -\left[x^2\left(x^2-2x+1\right)+4x\left(x^2-2x+1\right)+4\left(x^2-2x+1\right)\right] \\ &= -\left[x^4-2x^3+x^2+4x^3-8x^2+4x+4x^2-8x+4\right] \\ &= -\left[x^4+2x^3-3x^2-4x+4\right] \\ &= -x^4-2x^3+3x^2+4x-4 \end{aligned}$$

Thus, the leading term of $f(x)$ is $-x^4$, which has degree 4 and coefficient of -1.

Therefore, since the degree is even and the coefficient is negative, both ends of the graph point down.

Find the zeros.

$$-(x+2)^2(x-1)^2 = 0$$

$$(x+2)^2 = 0 \quad \text{or} \quad (x-1)^2 = 0$$

$$x+2 = 0 \qquad x-1=0$$

$$x=-2 \qquad x=1$$

So, the graph has zeros (x-intercepts) at 1 and -2. Now determine if each x-intercept is a turning point by examining the zero's multiplicity. The multiplicity of both zeros is 2 since the exponent of both related factors is 2. Since the multiplicity is even, the curve has a turning point at both x-intercepts.

Find the y-intercept. $f(0) = -(0+2)^2(0-1)^2 = -(2)^2(-1)^2 = -4$ So, the y-intercept of $f(x)$ is at -4.

Use substitution to find several additional points on the graph near where $x=-2$, $x=0$, and $x=1$.

x	$-(x+2)^2(x-1)^2$	$f(x)$	$(x,\ f(x))$
-3	$-(-3+2)^2(-3-1)^2$	-16	$(-3,\ -16)$
-1	$-(-1+2)^2(-1-1)^2$	-4	$(-1,\ -4)$
2	$-(2+2)^2(2-1)^2$	-16	$(2,\ -16)$

Plot those three points, along with the y-intercept and the x-intercepts. Then, sketch a curve that points down to the far left, goes through $(-3,\ -16)$, and then turns at $(-2,\ 0)$ and passes through $(-1,\ -4)$, and then turns and passes through $(0,\ -4)$, and continues up and then turns at $(1,\ 0)$, then continues down to the far right, passing through $(2,\ -16)$.

This concept is covered in Thinkwell's College Algebra lecture "Graphing Polynomial Functions: Another Example."

10. **Answer:** $\begin{cases} \mathbf{y \le -x+1} \\ \mathbf{x^2+y^2 \ge 16} \end{cases}$

Explanation

The graph contains two figures, a line and a circle. Begin by writing the equation of the line.

The line intersects the y-axis at 1. Therefore, the y-intercept is 1. The line has a slope of -1. So, substitute $m=-1$ and $b=1$ into the slope-intercept form of a linear equation, $y=mx+b$, to write the equation of the line.

$y=mx+b \;\Rightarrow\; y=(-1)x+1 \;\Rightarrow\; y=-x+1$ Thus, the equation of the line is $y=-x+1$.

Notice that the shading is below the line and that the line is not drawn as a dashed line. Therefore, the linear inequality shown in this graph is $y \le -x+1$. Now write the equation of the circle. The circle is centered at the origin and the radius is 4. The general form of the equation of a circle centered at the origin is $x^2+y^2=r^2$, where r is the circle's radius.

So, the equation of the circle is $x^2+y^2=4^2$, or $x^2+y^2=16$. Notice that the shading is outside of the circle and the circle is not drawn with a dashed line. Therefore, the inequality is $x^2+y^2 \ge 16$.

So, the system of inequalities shown in the graph is $y \le -x+1$ and $x^2+y^2 \ge 16$.

This concept is covered in Thinkwell's College Algebra lecture "Graphing the Solution Set of a System of Inequalities."

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