

## Thinkwell's Placement Test 6 Answer Key

If you answered 7 or more Test 6 questions correctly, we recommend Thinkwell's Precalculus. If you answered fewer than 7 Test 6 questions correctly, we recommend Thinkwell's College Algebra (Algebra 2).

### 1. Answer: -3

#### Explanation

The given function  $f(x)$  is a piecewise function.

$$f(x) = \begin{cases} -2x & \text{if } x < -2 \\ 5x + 7 & \text{if } -2 \leq x \leq 4 \\ -7x & \text{if } x > 4 \end{cases}$$

The domain is divided into three parts:  $x$ -values less than  $-2$ ,  $x$ -values between  $-2$  and  $4$  (including  $-2$  and  $4$ ), and  $x$ -values greater than  $4$ . For all  $x$ -values less than  $-2$ , the function used is  $f(x) = -2x$ . For all  $x$ -values between  $-2$  and  $4$ , the function used is  $f(x) = 5x + 7$ . And, for all values greater than  $4$ , the function used is  $f(x) = -7x$ . Therefore, the second function,  $f(x) = 5x + 7$ , is used to evaluate  $f(-2)$ , since  $-2$  is in the domain of this piece of the function.  $f(-2) = 5(-2) + 7 = -3$ . Thus,  $f(-2) = -3$ .

This concept is covered in Thinkwell's College Algebra topic "Evaluating Piecewise-Defined Functions for Given Values."

### 2. Answer: vertex (3, 2); $x = 3$

#### Explanation

The vertex of a parabola defined by a quadratic equation,  $y = ax^2 + bx + c$ , is located at the point  $(h, k)$ , where  $h = -\frac{b}{2a}$  and  $k = ah^2 + bh + c$ . So, to find the coordinates of a parabola's vertex, first find the value of  $h$ , and then substitute that  $h$ -value into the given equation to find the value of  $k$ . The given equation,  $y = 5x^2 - 30x + 47$ , is in the standard form of a quadratic equation,  $y = ax^2 + bx + c$ , where  $a = 5$ ,  $b = -30$ , and  $c = 47$ .

Use the values of  $a$  and  $b$  to find the  $x$ -coordinate of the vertex,  $h$ .  $h = -\frac{b}{2a} = -\frac{-30}{2(5)} = 3$  So, the vertex's  $x$ -coordinate is 3.

Now substitute 3 into the given equation to find the value of  $k$ , which is the vertex's  $y$ -coordinate.

$$\begin{aligned} y &= 5x^2 - 30x + 47 \\ &= 5(3)^2 - 30(3) + 47 && \text{Substitute 3 for } x. \\ &= 5(9) - 30(3) + 47 && \text{Evaluate the power.} \\ &= 2 && \text{Simplify.} \end{aligned}$$

So, the vertex's  $y$ -coordinate is 2.

Therefore, the parabola's vertex is at  $(3, 2)$ .

The axis of symmetry of a vertically opening parabola is a vertical line through the parabola's vertex. Since it is the  $x$ -variable that is squared in the given equation, the parabola opens vertically. Therefore, the axis of symmetry is the vertical line that passes through the vertex,  $(3, 2)$ . The equation of the vertical line that passes through  $(3, 2)$  is  $x = 3$ . Thus, the axis of symmetry is the line  $x = 3$ .

This concept is covered in Thinkwell's College Algebra topic "Determining Information about a Parabola from Its Equation."

### 3. Answer: $x = -\frac{1}{32}$ or $x = 1$

#### Explanation

Notice that the given equation,  $2x^{\frac{2}{5}} - x^{\frac{1}{5}} - 1 = 0$ , can be written in quadratic form if a variable is substituted for  $x^{\frac{1}{5}}$ . Let  $g = x^{\frac{1}{5}}$ . Substitute  $g$  into the equation and then solve the resulting quadratic equation for  $g$  by factoring (or by using the quadratic formula).

After the new equation is solved for  $g$ , substitute  $x^{\frac{1}{5}}$  for  $g$  and solve for  $x$ .

$$\begin{aligned} 2x^{\frac{2}{5}} - x^{\frac{1}{5}} - 1 &= 0 \\ 2x^{\frac{1}{5} \cdot 2} - x^{\frac{1}{5}} - 1 &= 0 && \text{Write the exponent } 2/5 \text{ as a product where one factor is } 1/5. \\ 2\left(x^{\frac{1}{5}}\right)^2 - x^{\frac{1}{5}} - 1 &= 0 && \text{Product of Powers Property} \\ 2g^2 - g - 1 &= 0 && \text{Substitute } g \text{ for } x^{\frac{1}{5}}. \\ (2g + 1)(g - 1) &= 0 && \text{Factor.} \\ 2g + 1 = 0 & \quad \text{or} \quad g - 1 = 0 && \text{Zero Factor Property} \\ g = -1/2 & \quad g = 1 && \text{Solve for } g. \\ x^{\frac{1}{5}} = -1/2 & \quad \text{or} \quad x^{\frac{1}{5}} = 1 && \text{Substitute } x^{\frac{1}{5}} \text{ for } g. \\ x = -1/32 & \quad x = 1 && \text{Solve for } x. \end{aligned}$$

This concept is covered in Thinkwell's College Algebra topic "Solving Fancy Quadratics."

4. **Answer:**  $y = 9$

Explanation

Square each side and simplify.

$$\begin{aligned}\sqrt{2y+7} &= \sqrt{2y-9} + 2 \\ (\sqrt{2y+7})^2 &= (\sqrt{2y-9} + 2)^2 && \text{Square both sides.} \\ 2y+7 &= (\sqrt{2y-9} + 2)(\sqrt{2y-9} + 2) && \text{Expand the power on the right side.} \\ 2y+7 &= (2y-9) + 4\sqrt{2y-9} + 4 && \text{FOIL} \\ 2y+7 &= 2y-5 + 4\sqrt{2y-9} && \text{Combine like terms.} \\ 12 &= 4\sqrt{2y-9} && \text{Subtract } 2y \text{ from each side and add } 5 \text{ to each side.} \\ 3 &= \sqrt{2y-9} && \text{Divide each side by } 4. \\ (3)^2 &= (\sqrt{2y-9})^2 && \text{Square both sides.} \\ 9 &= 2y-9 && \text{Simplify the powers.} \\ 18 &= 2y && \text{Add } 9 \text{ to each side.} \\ y &= 9 && \text{Divide each side by } 2.\end{aligned}$$

Now isolate the remaining radical and then square both sides again.

This concept is covered in Thinkwell's College Algebra topic "Solving an Equation with Two Radicals."

5. **Answer:**  $\frac{w^2 - 2w - 1}{w^2 + 3w + 7}$

Explanation

Factor the denominators.

$$\frac{\frac{1}{w^2 - 3w - 4} + \frac{1}{w + 3}}{\frac{1}{w - 4} - \frac{1}{w^2 + 4w + 3}} = \frac{\frac{1}{(w - 4)(w + 1)} + \frac{1}{w + 3}}{\frac{1}{w - 4} - \frac{1}{(w + 3)(w + 1)}}$$

The common denominator of the four terms is  $(w - 4)(w + 1)(w + 3)$ , so multiply the expression by  $\frac{(w - 4)(w + 1)(w + 3)}{(w - 4)(w + 1)(w + 3)}$ .

$$\begin{aligned}\frac{(w - 4)(w + 1)(w + 3)}{(w - 4)(w + 1)(w + 3)} \left( \frac{\frac{1}{(w - 4)(w + 1)} + \frac{1}{w + 3}}{\frac{1}{w - 4} - \frac{1}{(w + 3)(w + 1)}} \right) &= \frac{\frac{1(w - 4)(w + 1)(w + 3)}{(w - 4)(w + 1)} + \frac{1(w - 4)(w + 1)(w + 3)}{w + 3}}{\frac{1(w - 4)(w + 1)(w + 3)}{w - 4} - \frac{1(w - 4)(w + 1)(w + 3)}{(w + 3)(w + 1)}} \\ &= \frac{(w + 3) + (w - 4)(w + 1)}{(w + 1)(w + 3) - (w - 4)} \\ &= \frac{w + 3 + w^2 - 3w - 4}{w^2 + 4w + 3 - w + 4} \\ &= \frac{w^2 - 2w - 1}{w^2 + 3w + 7}\end{aligned}$$

This concept is covered in Thinkwell's College Algebra topic "Rewriting Complex Fractions."

6. Answer:  $\frac{1 \pm \sqrt{1 - k^2 q^2}}{k}$

Explanation

Multiply both sides of the equation by the expression in the denominator,  $p^2 + q^2$ . Then, set the equation equal to 0.

$$k = \frac{2p}{p^2 + q^2}$$

$$k(p^2 + q^2) = \frac{2p}{p^2 + q^2}(p^2 + q^2) \quad \text{Multiply both sides by } p^2 + q^2.$$

$$kp^2 + kq^2 = 2p \quad \text{Simplify.}$$

$$kp^2 - 2p + kq^2 = 0 \quad \text{Set equal to zero.}$$

Notice that the equation is quadratic in terms of  $p$ ,  $kp^2 - 2p + kq^2 = 0$ . So, use the quadratic formula to solve for  $p$ . Identify the values of  $a$ ,  $b$ , and  $c$ .  $kp^2 - 2p + kq^2 = 0 \Rightarrow a = k, b = -2$ , and  $c = kq^2$

$$p = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(k)(kq^2)}}{2(k)} \quad \text{Substitute } k, -2, \text{ and } kq^2 \text{ for } a, b, \text{ and } c \text{ in the quadratic formula.}$$

$$= \frac{2 \pm \sqrt{4 - 4k^2 q^2}}{2k} \quad \text{Simplify.}$$

$$= \frac{2 \pm \sqrt{4(1 - k^2 q^2)}}{2k} \quad \text{Factor the radicand.}$$

$$= \frac{2 \pm 2\sqrt{1 - k^2 q^2}}{2k} \quad \text{Simplify the radical.}$$

$$= \frac{1 \pm \sqrt{1 - k^2 q^2}}{k} \quad \text{Remove the factor of 2.}$$

This concept is covered in Thinkwell's College Algebra topic "Solving for a Squared Variable."

7. Answer:  $-2x + 1, R(2)$

Explanation

Since the denominator does not include an  $x$ -term, add the placeholder  $0x$  to the denominator. Use long division to divide.

$$\frac{6x^3 - 3x^2 - 2x + 3}{-3x^2 + 1} = \frac{6x^3 - 3x^2 - 2x + 3}{-3x^2 + 0x + 1} \quad \begin{array}{r} -2x + 1 \\ -3x^2 + 0x + 1 \overline{) 6x^3 - 3x^2 - 2x + 3} \\ \underline{-(6x^3 + 0x^2 - 2x)} \\ -3x^2 + 0x + 3 \\ \underline{-(-3x^2 + 0x + 1)} \\ 2 \end{array}$$

Thus, the quotient is  $-2x + 1, R(2)$ .

This concept is covered in Thinkwell's College Algebra topic "Long Division: Another Example."

8. Answer:  $x = -4$  with multiplicity three;  $x = 3$  with multiplicity one

Explanation

Since  $x = -4$  is a zero,  $x + 4$  is a factor. Use synthetic division to divide the polynomial by the factor  $x + 4$ .

$$\begin{array}{r|rrrrr} -4 & 1 & 9 & 12 & -80 & -192 \\ & & -4 & -20 & 32 & 192 \\ \hline & 1 & 5 & -8 & -48 & 0 \end{array} \quad \text{So, } \frac{x^4 + 9x^3 + 12x^2 - 80x - 192}{x + 4} = x^3 + 5x^2 - 8x - 48.$$

It follows that  $x^4 + 9x^3 + 12x^2 - 80x - 192 = (x + 4)(x^3 + 5x^2 - 8x - 48)$ .

Since  $x = 3$  is a zero,  $x - 3$  is a factor. Use synthetic division to divide  $x^3 + 5x^2 - 8x - 48$  by  $x - 3$ .

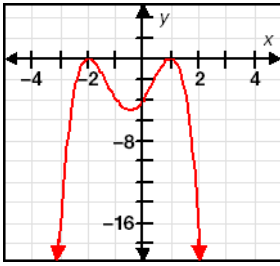
$$\begin{array}{r|rrrr} 3 & 1 & 5 & -8 & -48 \\ & & 3 & 24 & 48 \\ \hline & 1 & 8 & 16 & 0 \end{array} \quad \text{So, } \frac{x^3 + 5x^2 - 8x - 48}{x - 3} = x^2 + 8x + 16.$$

It follows that  $x^4 + 9x^3 + 12x^2 - 80x - 192 = (x + 4)(x^3 + 5x^2 - 8x - 48) = (x + 4)(x - 3)(x^2 + 8x + 16)$ .

Factor  $x^2 + 8x + 16$ .  $(x + 4)(x - 3)(x^2 + 8x + 16) = (x + 4)(x - 3)(x + 4)(x + 4) = (x + 4)^3(x - 3)$

Thus, the zeros are  $x = -4$  with multiplicity three and  $x = 3$  with multiplicity one.

This concept is covered in Thinkwell's College Algebra topic "Finding the Zeros of a Polynomial from Start to Finish."



9. **Answer:**

Explanation

Expand the polynomial.

$$\begin{aligned}
 f(x) &= -(x+2)^2(x-1)^2 \\
 &= -(x^2 + 4x + 4)(x^2 - 2x + 1) \\
 &= -[x^2(x^2 - 2x + 1) + 4x(x^2 - 2x + 1) + 4(x^2 - 2x + 1)] \\
 &= -[x^4 - 2x^3 + x^2 + 4x^3 - 8x^2 + 4x + 4x^2 - 8x + 4] \\
 &= -[x^4 + 2x^3 - 3x^2 - 4x + 4] \\
 &= -x^4 - 2x^3 + 3x^2 + 4x - 4
 \end{aligned}$$

Thus, the leading term of  $f(x)$  is  $-x^4$ , which has degree 4 and coefficient of  $-1$ .

Therefore, since the degree is even and the coefficient is negative, both ends of the graph point down.

Find the zeros.

$$\begin{aligned}
 -(x+2)^2(x-1)^2 &= 0 \\
 (x+2)^2 &= 0 \quad \text{or} \quad (x-1)^2 = 0 \\
 x+2 &= 0 \quad \quad \quad x-1 = 0 \\
 x &= -2 \quad \quad \quad x = 1
 \end{aligned}$$

So, the graph has zeros ( $x$ -intercepts) at  $1$  and  $-2$ . Now determine if each  $x$ -intercept is a turning point by examining the zero's multiplicity. The multiplicity of both zeros is  $2$  since the exponent of both related factors is  $2$ . Since the multiplicity is even, the curve has a turning point at both  $x$ -intercepts.

Find the  $y$ -intercept.  $f(0) = -(0+2)^2(0-1)^2 = -(2)^2(-1)^2 = -4$  So, the  $y$ -intercept of  $f(x)$  is at  $-4$ .

Use substitution to find several additional points on the graph near where  $x = -2$ ,  $x = 0$ , and  $x = 1$ .

$x$	$-(x+2)^2(x-1)^2$	$f(x)$	$(x, f(x))$
$-3$	$-(-3+2)^2(-3-1)^2$	$-16$	$(-3, -16)$
$-1$	$-(-1+2)^2(-1-1)^2$	$-4$	$(-1, -4)$
$2$	$-(2+2)^2(2-1)^2$	$-16$	$(2, -16)$

Plot those three points, along with the  $y$ -intercept and the  $x$ -intercepts. Then, sketch a curve that points down to the far left, goes through  $(-3, -16)$ , and then turns at  $(-2, 0)$  and passes through  $(-1, -4)$ , and then turns and passes through  $(0, -4)$ , and continues up and then turns at  $(1, 0)$ , then continues down to the far right, passing through  $(2, -16)$ .

*This concept is covered in Thinkwell's College Algebra topic "Graphing Polynomial Functions: Another Example."*

10. **Answer:** 
$$\begin{cases} y \leq -x + 1 \\ x^2 + y^2 \geq 16 \end{cases}$$

Explanation

The graph contains two figures, a line and a circle. Begin by writing the equation of the line.

The line intersects the  $y$ -axis at  $1$ . Therefore, the  $y$ -intercept is  $1$ . The line has a slope of  $-1$ . So, substitute  $m = -1$  and  $b = 1$  into the slope-intercept form of a linear equation,  $y = mx + b$ , to write the equation of the line.

$$y = mx + b \Rightarrow y = (-1)x + 1 \Rightarrow y = -x + 1 \quad \text{Thus, the equation of the line is } y = -x + 1.$$

Notice that the shading is below the line and that the line is not drawn as a dashed line. Therefore, the linear inequality shown in this graph is  $y \leq -x + 1$ . Now write the equation of the circle. The circle is centered at the origin and the radius is  $4$ .

The general form of the equation of a circle centered at the origin is  $x^2 + y^2 = r^2$ , where  $r$  is the circle's radius.

So, the equation of the circle is  $x^2 + y^2 = 4^2$ , or  $x^2 + y^2 = 16$ . Notice that the shading is outside of the circle and the circle is not drawn with a dashed line. Therefore, the inequality is  $x^2 + y^2 \geq 16$ .

So, the system of inequalities shown in the graph is  $y \leq -x + 1$  and  $x^2 + y^2 \geq 16$ .

*This concept is covered in Thinkwell's College Algebra topic "Graphing the Solution Set of a System of Inequalities."*

### Guidelines for Interpreting Placement Test Scores

Placement Test	Number of Correct Answers	Recommendation
Placement Test 1	5 or more	<a href="#">Thinkwell's 6<sup>th</sup> Grade Math</a>
Placement Test 2	6 or less	<a href="#">Thinkwell's 6<sup>th</sup> Grade Math</a>
	7 or more	<a href="#">Thinkwell's 7<sup>th</sup> Grade Math</a>
Placement Test 3	4 or less	<i>complete Placement Test 2</i>
	5 or 6	<a href="#">Thinkwell's 7<sup>th</sup> Grade Math</a>
	7 or more	<a href="#">Thinkwell's 8<sup>th</sup> Grade Math</a>
Placement Test 4	4 or less	<i>complete Placement Test 3</i>
	5 or 6	<a href="#">Thinkwell's 8<sup>th</sup> Grade Math</a>
	7 or more	<a href="#">Thinkwell's Intermediate Algebra (Algebra 1)</a>
Placement Test 5	4 or less	<i>complete Placement Test 4</i>
	5 or 6	<a href="#">Thinkwell's Intermediate Algebra (Algebra 1)</a>
	7 or more	<a href="#">Thinkwell's College Algebra (Algebra 2)</a>
Placement Test 6	4 or less	<i>complete Placement Test 5</i>
	5 or 6	<a href="#">Thinkwell's College Algebra (Algebra 2)</a>
	7 or more	<a href="#">Thinkwell's Precalculus</a>
Placement Test 7	4 or less	<i>complete Placement Test 6</i>
	5 or 6	<a href="#">Thinkwell's Precalculus</a>
	7 or more	<a href="#">Thinkwell's Calculus</a>