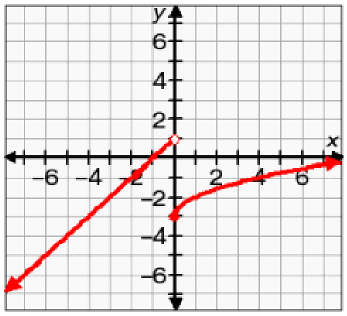
Thinkwell's Placement Test 7 Answer Key

If you answered 7 or more Test 7 questions correctly, we recommend Thinkwell's Calculus. If you answered fewer than 7 Test 7 questions correctly, we recommend Thinkwell's Precalculus.

1. **Answer:** 

Explanation

To graph the given function, $f(x)=\begin{cases}\sqrt{x}-3, & x\geq 0\\ x+1, & x<0\end{cases}$, graph each piece over its given domain.

The first function, $f(x)=\sqrt{x}-3$, is a curve through the points $(0,\ -3)$ and $(1,\ -2)$. The domain is $x\geq 0$, so the curve has a closed circle at $(0,\ -3)$ and extends infinitely in the positive direction.

The second function, $f(x)=x+1$, is a line with slope 1 and y-intercept 1. The domain is $x<0$, so the line has an open circle at $(0,\ 1)$ and extends infinitely in the negative direction.

This concept is covered in Thinkwell's Precalculus topic "Graphing Piecewise-Defined Functions."

2. **Answer: –2*i*, 2*i*, –1, 5**

Explanation

The function $P(x)$ has all real coefficients. Therefore, by the Conjugate Pair Theorem, if $2i$ is a zero, then its conjugate $-2i$ must also be a zero. Next, write the two known zeros as factors.

zero	corresponding factor
$2i$	$x-2i$
$-2i$	$x-(-2i)=x+2i$

Multiply the factors.

$$(x-2i)(x+2i)$$
$$x^2+2ix-2ix-4i^2$$
$$x^2+4$$

Divide $P(x)$ by this product.

$$\begin{array}{r} x^2-4x-5 \\ x^2+4\overline{\big)\,x^4-4x^3-x^2-16x-20} \\ -(x^4 \qquad +4x^2) \\ \hline -4x^3-5x^2-16x \\ -(-4x^3 \qquad -16x) \\ \hline -5x^2 \qquad -20 \\ -(-5x^2 \qquad -20) \\ \hline 0 \end{array}$$

The remainder is 0, so x^2-4x-5 is a factor of $P(x)$.

Solve $x^2-4x-5=0$ to find the zeros of $P(x)$.

$$x^2-4x-5=0$$
$$(x+1)(x-5)=0$$
$$x=-1 \text{ or } x=5$$

Thus, -1 and 5 are zeros of $P(x)$.

Therefore, the zeros are $-2i$, $2i$, -1, and 5.

This concept is covered in Thinkwell's Precalculus topic "The Conjugate Pair Theorem."

3. **Answer:** $\dfrac{\frac{19}{4}}{x+2}+\dfrac{\frac{17}{4}}{x}-\dfrac{\frac{3}{2}}{x^2}$

Explanation

Express as a partial fraction with A, B, and C as the numerators.

$$\frac{9x^2+7x-3}{x^3+2x^2}=\frac{9x^2+7x-3}{x^2(x+2)}=\frac{A}{x+2}+\frac{B}{x}+\frac{C}{x^2}$$

Create a common denominator and simplify the numerator. Then, factor out x^2 and x.

$$\frac{A}{x+2}+\frac{B}{x}+\frac{C}{x^2}=\frac{A(x^2)+B(x+2)(x)+C(x+2)}{(x+2)(x^2)}$$
$$=\frac{Ax^2+Bx^2+2Bx+Cx+2C}{(x+2)(x^2)}$$
$$=\frac{(A+B)x^2+(2B+C)x+(2C)}{(x+2)(x^2)}$$

Equate the coefficients to write a system of equations.

$$\frac{9x^2+7x-3}{x^3+2x^2}=\frac{(A+B)x^2+(2B+C)x+(2C)}{(x+2)(x^2)}\Rightarrow\begin{cases}A+B=9\\2B+C=7\\2C=-3\end{cases}$$

Solve for C.

$$2C=-3$$
$$C=-\frac{3}{2}$$

Solve for B.

$$2B+C=7$$
$$2B+\left(-\frac{3}{2}\right)=7$$
$$B=\frac{17}{4}$$

Solve for A.

$$A+B=9$$
$$A+\frac{17}{4}=9$$
$$A=\frac{19}{4}$$

Substitute to find the partial decomposition.

$$\frac{9x^2+7x-3}{x^3+2x^2}=\frac{A}{x+2}+\frac{B}{x}+\frac{C}{x^2}=\frac{\frac{19}{4}}{x+2}+\frac{\frac{17}{4}}{x}-\frac{\frac{3}{2}}{x^2}$$

This concept is covered in Thinkwell's Precalculus topic "Partial Fractions: Another Example."

4. **Answer: 15/4**

Explanation

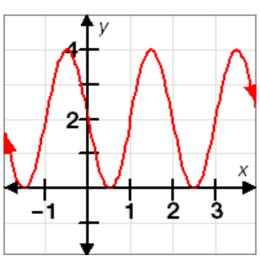
Recall the values of the trigonometric functions for the angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$: $\sin\frac{\pi}{6}=\frac{1}{2}$, $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, and $\tan\frac{\pi}{3}=\sqrt{3}$.

Use these values to simplify the expression.

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{4}+\tan^2\frac{\pi}{3}$$
$$\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2+\left(\sqrt{3}\right)^2$$
$$\frac{1}{4}+\frac{2}{4}+3$$
$$\frac{15}{4}$$

This concept is covered in Thinkwell's Precalculus topic "Trigonometric Functions of Important Angles."

5. **Answer:**



Explanation

Begin by graphing the function $y=2\sin\pi x$, which has a period of 2 and an amplitude of 2.
Then, to graph $y=2\sin\pi(x-1)+2$, shift the graph of $y=2\sin\pi x$ right 1 unit and up 2 units.

This concept is covered in Thinkwell's Precalculus topic "Graphing Sine and Cosine Functions with Phase Shifts."

6. **Answer:** $\mathbf{4\sec^2\theta}$

Explanation

Subtract the fractions by using a common denominator and then use the Pythagorean Identity, $\sin\theta^2+\cos\theta^2=1$, to simplify.

$$\frac{2}{\sin\theta+1}-\frac{2}{\sin\theta-1}$$

$$\frac{2(\sin\theta-1)-2(\sin\theta+1)}{(\sin\theta-1)(\sin\theta+1)}$$ *Write the fractions with a common denominator and subtract.*

$$\frac{2\sin\theta-2-(2\sin\theta+2)}{\sin\theta^2-1}$$ *Distribute in the numerator twice and FOIL in the denominator.*

$$\frac{2\sin\theta-2-2\sin\theta-2}{\sin\theta^2-1}$$ *Distribute the negative in the numerator.*

$$\frac{-4}{\sin\theta^2-1}$$ *Combine like terms in the numerator.*

$$\frac{-4}{\sin\theta^2-\left(\sin\theta^2+\cos\theta^2\right)}$$ *Use the Pythagorean Identity.*

$$\frac{-4}{-\cos\theta^2}$$ *Combine like terms in the denominator.*

$$4\sec^2\theta$$ *Simplify using 1/cosθ = secθ.*

This concept is covered in Thinkwell's Precalculus topic "Simplifying Products of Binomials Involving Trigonometric Functions."

7. **Answer:** $\boldsymbol{\theta=0,\frac{\pi}{6},\frac{5\pi}{6}}$**, and** $\boldsymbol{\pi}$

Explanation

Manipulate the equation so that it is equal to 0 and then factor.

$$\sin^2\theta=\frac{1}{2}\sin\theta$$

$$\sin^2\theta-\frac{1}{2}\sin\theta=0$$

$$\sin\theta\left(\sin\theta-\frac{1}{2}\right)=0$$

Now set each factor equal to zero and solve each equation.

$\boxed{\sin\theta=0}$

Sine has a value of zero when θ is equal to $0+\pi n$. So within the specified interval there are two solutions, $x=0$ and $x=\pi$.

$$\boxed{\begin{aligned}\sin\theta-\frac{1}{2}&=0\\ \sin\theta&=\frac{1}{2}\end{aligned}}$$

When sine takes a value of one-half, then the opposite side is equal to 1 and the hypotenuse is equal to 2. These numbers generate the famous $1{:}\sqrt{3}{:}2$ triangle, also known as the $30°:60°:90°$ triangle.

$$\theta=\frac{\pi}{6}\text{ and }\frac{5\pi}{6}$$

So, the solutions within the interval $[0,2\pi)$ are $\theta=0,\ \frac{\pi}{6},\ \frac{5\pi}{6}$, and π.

This concept is covered in Thinkwell's Precalculus topic "Solving Trigonometric Equations by Factoring."

8. **Answer: 1/25**

Explanation

Substitute 2 for $E(t)$ in the given function, $E(t) = 2\cos 50\pi t$, and solve for t.

$$2 = 2\cos 50\pi t$$
$$1 = \cos 50\pi t$$
$$\cos^{-1}(1) = 50\pi t$$
$$2\pi \cdot n = 50\pi t$$
$$t = \frac{1}{25}n, \text{ where } n \text{ is any integer}$$

The smallest possible value of t will occur when $n = 1$ and that value of t is $\frac{1}{25}$.

This concept is covered in Thinkwell's Precalculus topic "Solving Word Problems Involving Trigonometric Equations."

9. **Answer:** $\mathbf{\frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

Explanation

To solve the equation $\cos 2t = \cos t$ for t when t is in the interval $[0, 2\pi)$, use the double-angle formula for $\cos 2t$.

$$\cos 2t = \cos t$$
$$2\cos^2 t - 1 = \cos t \quad \textit{Double - Angle Formula}$$
$$2\cos^2 t - \cos t - 1 = 0 \quad \textit{Subtract cos t from each side.}$$
$$(2\cos t + 1)(\cos t - 1) = 0 \quad \textit{Factor.}$$

Set each factor equal to 0 and solve each equation for t.

$$2\cos t + 1 = 0 \qquad \cos t - 1 = 0$$
$$\cos t = -\frac{1}{2} \qquad \cos t = 1$$
$$t = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad t = 0$$

This concept is covered in Thinkwell's Precalculus topic "Using Double-Angle Identities."

10. **Answer: 13**

Explanation

Since $a = 7.25$ inches, and side b is 1.3 inches longer than side a, it follows that $b = 7.25 + 1.3 = 8.55$ inches. It is given that $\beta = 38°$. In relation to this known angle, β, the length of the adjacent side, a, is less than the length of the opposite side, b. So, exactly one triangle can be formed where $a = 7.25$ inches, $b = 8.55$ inches, and $\beta = 38°$, since $a \le b$.

Use the Law of Sines to find α.

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$$
$$\frac{\sin 38°}{8.55} = \frac{\sin\alpha}{7.25}$$
$$\sin\alpha = \frac{(7.25)(\sin 38°)}{8.55}$$
$$\sin\alpha = 0.522...$$
$$\alpha \approx 31.5°$$

Now use the measures of α and β, along with the fact that the sum of the angles of a triangle is 180°, to find γ.

$$\alpha + \beta + \gamma = 180°$$
$$31.5° + 38° + \gamma = 180°$$
$$69.5° + \gamma = 180°$$
$$\gamma = 110.5°$$

Use the Law of Sines again to find c.

$$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$$
$$\frac{\sin 110.5°}{c} = \frac{\sin 38°}{8.55}$$
$$c = \frac{(8.55)(\sin 110.5°)}{\sin 38°}$$
$$c = 13$$

This concept is covered in Thinkwell's Precalculus topic "Solving a Triangle (SAS): Another Example."

Guidelines for Interpreting Placement Test Scores

Placement Test	Number of Correct Answers	Recommendation
Placement Test 1	5 or more	Thinkwell's 6th Grade Math
Placement Test 2	6 or less	Thinkwell's 6th Grade Math
	7 or more	Thinkwell's 7th Grade Math
Placement Test 3	4 or less	*complete Placement Test 2*
	5 or 6	Thinkwell's 7th Grade Math
	7 or more	Thinkwell's 8th Grade Math
Placement Test 4	4 or less	*complete Placement Test 3*
	5 or 6	Thinkwell's 8th Grade Math
	7 or more	Thinkwell's Intermediate Algebra (Algebra 1)
Placement Test 5	4 or less	*complete Placement Test 4*
	5 or 6	Thinkwell's Intermediate Algebra (Algebra 1)
	7 or more	Thinkwell's College Algebra (Algebra 2)
Placement Test 6	4 or less	*complete Placement Test 5*
	5 or 6	Thinkwell's College Algebra (Algebra 2)
	7 or more	Thinkwell's Precalculus
Placement Test 7	4 or less	*complete Placement Test 6*
	5 or 6	Thinkwell's Precalculus
	7 or more	Thinkwell's Calculus